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A NEW
CALCULATING MACHINE,

WITH

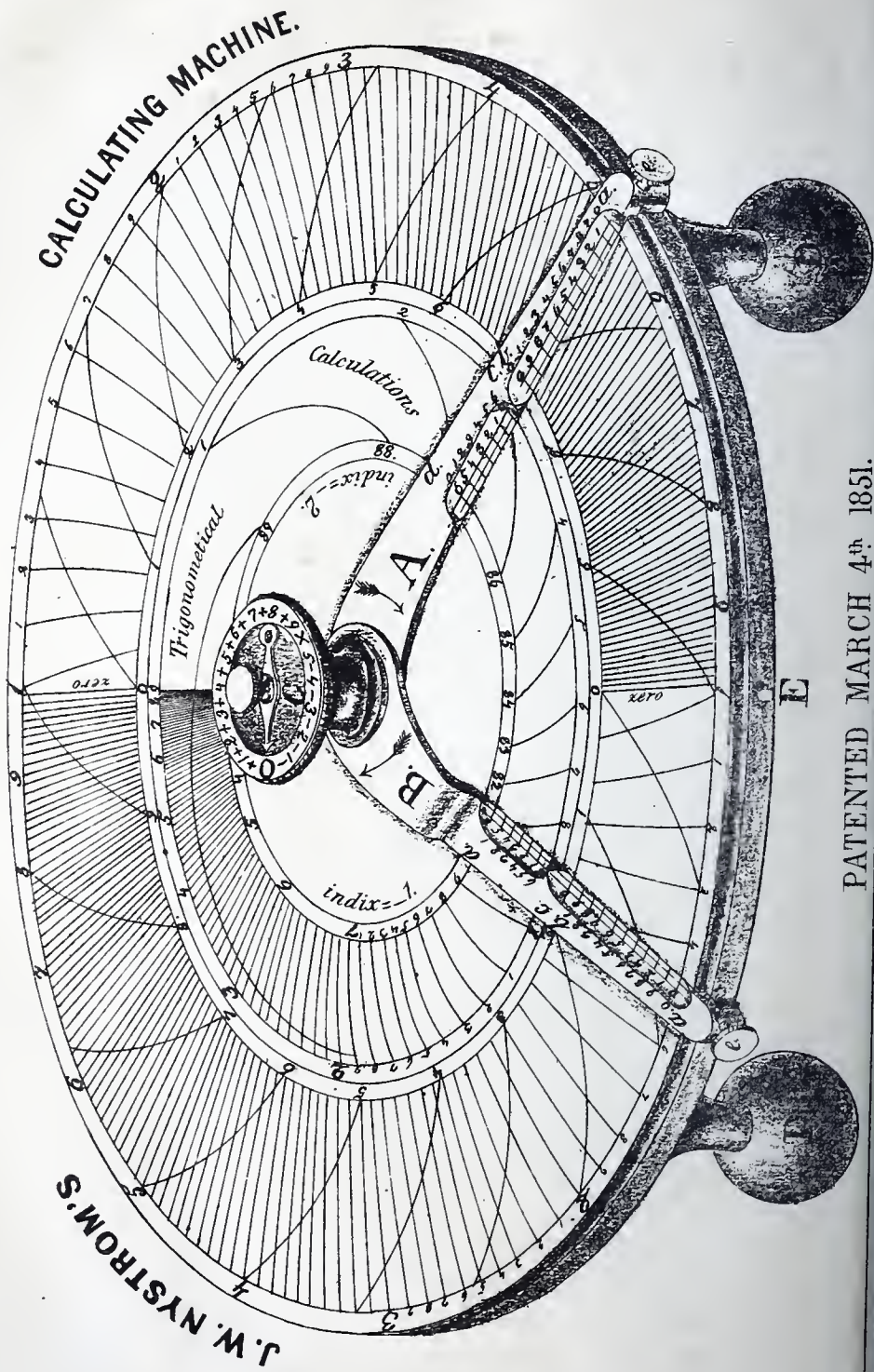
Practical Rules and Examples

FOR

USING THE SAME.

BY JOHN W. NYSTROM.

PHILADELPHIA:
R. W. BARNARD & SONS, PRINTERS.
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PREFACE.

IN this age of invention, the public is ever and anon startled by the announcement of some *time* or *labor saving machine*, whereby the old trodden paths of our fathers are forsaken, and we of this *go-ahead* age are enabled to accomplish in a day or an hour, what to them was the work of a lifetime.

In every department of life these rapid changes are being effected. All are interested in them. The grave Scholar, the Mathematician, and the Mechanic in his shop, have to acknowledge that machinery and skill lighten their toil and facilitate their operations.

The inventor of the Machine designed to accompany this book being a practical engineer, and having multiplied calculations to make, felt the necessity of some simple instrument to aid in calculation. After much thought and labor, he has succeeded in producing such an instrument—simple in construction, and portable in size, yet so complete that every variety of calculation may be computed by it, from the school-boy's easy sum to the most complicated astronomical and mathematical computation. Its operations are so simple, that it does not require great power of mind to comprehend them; they can be applied by any one to the common business purposes of life. The Engineer and Constructor, the Mathematician, the Surveyor, the Merchant, or the Teacher in the school room, will find this *Calculating Machine* to be all they desire. Its great recommendation is, the rapidity with which calculations can be made;—what, by the ordinary method, is the work of hours, filling pages with figures, can be done in a few seconds, without figures. Many important calculations are dispensed with, and often thrown aside before completion, owing to the length of time required to compute them.

For Teachers it will be found to be a valuable assistant, furnishing a test of the real acquirements of the pupil, without taxing the time of the teacher. Teachers are generally dependent upon text books for examples, particularly in the higher branches of mathematics, and it is easy for the pupil, knowing where the example comes from, to be furnished with an answer; but an ingenious teacher, furnished with one of these *Calculators*, may vary the examples *ad infinitum*, and have the answer almost in an instant before him. The pupil is thrown on his own resources for the proper solution of the problem.

For Surveyors it is invaluable, as the trigonometrical formula can be computed without any reference to tables of the trigonometrical lines, or logarithms, including navigation, as will be seen in examples in the latter part of the work, in Plain and Spherical Trigonometry.

In constructing this *Calculating Machine* a great difficulty presented itself, viz., the need of a correct instrument to draw the curved lines which are laid out in the logarithmic and trigonometrical divisions. Such an instrument has been completed, and one which can be relied on for its accuracy. The first division was done by the dividing machine in the office of the Coast Survey, at Washington, which is, perhaps, the best in the world. The facility with which the Machines can be drawn, enables the patentee to furnish them at greatly reduced prices.

The patentee flatters himself that his *Calculator* will recommend itself when fairly before the public, the want of such an instrument having been long felt. This Machine was invented in 1848, and received a *First Premium* at the Franklin Institute Exhibition in 1849. Since then, it has been patented, but other engagements have prevented the patentee from bringing it before the public until the present time.

J. W. NYSTROM, *Patentee*,

Philadelphia, January, 1852.

DESCRIPTION OF THE CALCULATING MACHINE.

THIS CALCULATING MACHINE consists of a round disk of metal or any other suitable material. It has two graduated arms, A and B, extending from the centre, C, to the periphery. On the outer end of each arm is a screw, *e*, for the purpose of fastening the arms in any particular place on the disk. In the centre is a screw, C, to clamp the two arms A and B together,—when clamped they can be moved around the centre, C.

The operation of calculation is performed by moving these two arms together and independently; for each operation the arm B must be set in one particular place on the disk, called *Zero*. In order to be correct and save time, and to facilitate the setting the arm B on *Zero*, there is placed on *Zero* a nail, E.

When the arm B approaches *Zero*, keep one finger on a spring attached to the nail E, under the disk; pinch the spring with the finger, the nail E will project over the periphery, and the arm B stops at *Zero*. A little practice, and this can be done with great facility.

On the disk are engraved 290 different curved lines; for each curved line are calculated 10 points; by joining these each curved line is obtained; that is, before the 290 curved lines can be drawn, 2900 points must be calculated. This, it will be seen, is a laborious undertaking, but patient industry accomplishes it, and by it a machine is obtained, on which any question of calculation can be computed, no matter how complicated it may be.

These curved lines are laid out in two concentric rings on the disk; on each edge of the two rings is a figure circle, which are marked with the letters *a*, *b*, *c*, and *d*. The curved lines which belong to the circle *a* are for *Multiplication* and *Division*, *Powers* and *Extracting of Roots*; circle *b* for *Addition* and *Subtraction*, or the *Logarithm* for the number in circle *a*.

Circle *a* is divided into a Logarithmic series, the distances between the curved lines being in proportion as the logarithm for the number represented by the curved lines (circle *a*).

Circle *b* is divided in an arithmetical series, where the difference between the terms is $=1$, and, by starting the two divisions of the circles *a* and *b* from a common point or radius, the circle *b* will be the logarithm

for circle *a*. It is not necessary that this commencing line should be in the direction of the radius, but on this instrument it is done so.

Before going further, it may be well to define what is meant by logarithm.

Logarithm is an exponent of a power, to which 10 must be raised to give a certain number. This exponent is called the *logarithm* for that number.

	Number.	Logarithm.	Exponent.		Number.
log.	100	= 2	because 10^2	=	100 square of 10.
log.	1000	= 3	" 10^3	=	1000 cube of 10.
log.	10000	= 4	" 10^4	=	10000 fourth power of 10.
log.	172800	= 5.2375	" $10^{5.2375}$	=	172800

Commonly expressed, 5.2375 is the logarithm for the number 172800. The unity of the logarithm is called the index, and the decimal part is called the mantissa. For the number =172800 we have—

$$\begin{array}{rcl}
 \text{Index,} & . & = 5 \\
 \text{Mantissa,} & . & = 0.2375 \\
 \hline
 \text{Logarithm,} & . & = 5.2375
 \end{array}$$

The invariable number 10 is called the *base* for the system of logarithms; it is not necessary that the base should be 10, it could be any number; but as our system of arithmetic has 10 to the base, it is more convenient to have the same base in the system of logarithms. All the tables of logarithms now in common use, are calculated with 10 to the base.*

But whatever the base may be, the nature of the logarithms remain the

* The question may be asked: Would there be any advantage in any other *common base* for the systems of Arithmetic and Logarithms? If we take 8 or 16 to the base, the system of Arithmetic would, in the first instance, be two figures shorter, as 1, 2, 3, 4, 5, 6, 7, 10; but if 16 be the base, six new figures must be added. To obtain six new figures, suppose we turn the old figures up and down, then the new Arithmetical system would be as follows: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16. This turning up and down of the last figures would be found a great aid in mental calculation, and the system would render it so easy that, for common business purposes, no figures need be written in their solution. All over the world, there is a strong effort being made to obtain the unity of all measures *equally* divided, viz., to divide the unity in 10ths and 100ths, called the decimal system. This is objected to, for the reason that the base in the Arithmetical system cannot be evenly divided by 4, 8, 16, &c.; yet it has been adopted in France, and partly in other countries. Even in France the preference is for the old system, but as it is against the law, the bargain is made in the old, and the bill in the new system. But if we had an arithmetic system of 16 to the base, the unity would at once be equally divided over the whole world.

same; as, the logarithm for the base = 1, and the logarithm of 1 = 0. Logarithms are convenient in *Multiplication, Division, Powers, and Extraction of Roots.*

Multiplication.—Add together the logarithms of the factors; the sum is the logarithm for the product; as

$$\begin{array}{rcl} 3456 \times 50. & \log. & 3456 = 3.53857 \\ & \log. & 50 = 1.69897 \end{array}$$

$$\text{Sum} = \underline{\quad\quad\quad} = 5.23754 = \log. 172800, \text{ the product.}$$

Division.—Subtract the logarithm for the divisor from the logarithm for the dividend; the difference is the logarithm for the quotient.

$$\begin{array}{rcl} 172800 : 50. & \log. & 172800 = 5.23754 \\ & \log. & 50 = 1.69897 \end{array}$$

$$\text{Difference} = \underline{\quad\quad\quad} = 3.53857 = \log. 3456, \text{ the quotient.}$$

Powers.—Multiply the logarithm for the number by the exponent of the power; the product is the logarithm for the power of the number; as 150^3 . $3 \times \log. 150 = 3 \times 2.176091 = 6.528273 = \log. 3375000 = 150^3$.

Extraction of Roots.—Divide the logarithm for the number by the index for the root; the quotient is the logarithm for the root of the number; as

$$\begin{array}{l} \sqrt[3]{3375000}. \log. 3375000 = 6.528273 : 3 = 2.176091 = \log. 150 \\ \qquad\qquad\qquad = \sqrt[3]{3375000}. \end{array}$$

This is the principle upon which the *Calculating Machine* is based, and the addition and subtraction of the logarithms of the numbers are computed without noticing the logarithm; only in moving the arms A and B together and independently between the numbers, by the intersection of the curved lines the result is obtained. It is not necessary to understand the nature of logarithms to be able to use this instrument; it requires only attention to the Rules given.

Circle a.—On circle *a* are two sizes of figures, of which the larger one represents the *first* figure in a number, and the small one the *second*; for instance, 28 is a number composed of two figures; 2 is the first figure, 8 the second.

Example. To set the arm A on number 28 (circle *a*). Set the arm A on the large 2, (circle *a*,) move the arm further until it comes to the small 8; (the eighth curved line from the large 2;) fasten the arm A with the screw *e*; the arm is then set on 28. But this 28 can represent any multiple or divisor of 10; for instance, 0.0028, 0.28, 2.8, 28, 280, 28000, &c.

If the number contains more than two figures, as 2839, the third and fourth figures are to be found on the arm where it intersects the curved

line; the third figure is the small figure on the arms, (counted from a), and the fourth figure the small divisions between the figures on the arms.

Circle b. Circle b is the logarithm for circle a ; that is, when the arm is set on a number on circle a , and at the same time the arm intersects a curved line belonging to the circle b , the intersection represents the decimal part of the logarithm (mantissa) for the number in circle a ; as, $\log. 7 = 0.845$. The first figure, 8, is the eighth curved line on circle b ; the second figure, 4, is the fourth figure on the arm, counted from b ; and the third figure, 5, is the fifth division between 4 and 5 on the arm, and 7 is the corresponding number on circle a . If it is $\log. 70 = 1.845$, the mantissa remains the same, but the index is $= 1$. It is well known that in the system of logarithms, the index is *always* one less than the number of figures in the number for which the logarithm is to be found; as, $\log. 7000 = 3.845$. Here the number is four figures, and the index $= 4 - 1 = 3$; but the mantissa is the same as for 7.

In higher calculations, where the number of figures in the result is uncertain, a correct account must be kept of the indices. For that purpose, a small hand is on the screw C, which is to be moved by the hand for each operation with the arms; as, for Multiplication, the indices are to be added. In Division and Subtraction, when the operations are finished, the small hand shows the index; add one to it, gives the number of figures in the result. If the index becomes negative, the result is a corresponding decimal fraction, and the hand shows how many 0 it is before the figures included in the unity 0. If the arm shows 28, (circle a), and the hand shows the

Index	=	—3	the decimal fraction is	0.0028
Index	=	—2	“ “	0.028
Index	=	—1	“ “	0.28
Index	=	0	the number is	2.8
Index	=	+1	“	28
Index	=	+3	“	2800
Index	=	+6	“	2800000

MULTIPLICATION.

Multiplication is computed on the circle a as follows:—

RULE I. Set the arm A on the first factor, fasten A^e ;^{*} set the arm B on Zero, fasten C; † loose A^e , and move the arms until the arm B comes to the second factor; then the arm A shows the product.

^{*} The arm A with the screw e .

† The arms A and B with the screw C.

Example 1. Multiply 3 by 2. Set the arm A on 3, (circle a ,) and B on *Zero*, fasten the arms with the screw C, and move them until the arm B comes to 2; then the arm A shows the product =6.

Example 2. Multiply 4 by 2. Set the arm A on 4, fasten A^e , set the arm B on *Zero*, fasten C, loose A^e , move the arms until B comes to 2; the arm A will show the product =8.

Example 3. Multiply 6 by 4. Set A on 6, fasten A^e , set B on *Zero*, fasten C, loose A^e , move the arms until B comes to 4, then A shows the product =24.

Example 4. Multiply 9 by 7. Set A on 9, fasten A^e , set B on *Zero*, fasten C, loose A^e , move the arms until B comes to 7; then A shows the product =63.

Example 5. Multiply 12 by 3. Set A on 12, (the second curved line from the large 1,) fasten A^e , set B on *Zero*, fasten C, loose A^e , move the arms until B comes to 3; then the arm A shows the product =36.

Example 6. Multiply 36 by 6. Set A on 36, fasten A^e , set B on *Zero*, fasten C, loose A^e , move the arms until B comes to 6; then the arm A shows the product =216. The third figure, 6, will be found on the arm where the curved line 1 intersects the same at figure 6 on the arm A, counted from a .

Example 7. Multiply 216 by 16. $216 \times 16 = 3456$.

Example 8. What is the area of a rectangle in square feet, when one side is 3 feet 6 inches, and the other 1 foot 9 inches? Set A on 3.5, B on *Zero*, fasten C, move the arms until B comes to 1.75; then the arm A shows the answer, 61.25 square feet.

Example 9. A pine board is 16 feet 3 inches long, and 9 inches wide; how many square feet are there in it? Set A on 16.25, B on *Zero*, fasten C, move the arms until B comes to 0.75, (9 inches = 0.75 feet;) then A shows the answer =12.19 square feet.

Example 10. How much will the same board cost at 4 cents per square foot? Set A on 12.19, B on *Zero*, fasten C, move the arms until B comes to 4; then A shows the answer =48.75 cents.

Example 11. How long is the periphery of a circle 15 inches in diameter? Set A on 15, B on *Zero*, fasten C, move the arms until B comes to 3.14; then A shows the answer =47.12 inches.

Example 12. The driving wheel on a locomotive is 5 feet 6 inches; how long is the periphery? Set A on 5.5, B on *Zero*, fasten C, move the arms until B comes to 3.14; then A shows the answer =17.27 feet.

Example 13. How much for 39 yards of ribbon, at 5 cents per yard? $39 \times 5 = \$1.95$. Set A on 39, B on *Zero*, fasten C, move the arms until B comes to 5; then A will show the answer = \$1.95.

Example 14. How many square yards are there in a piece of cloth 37 yards and 29 inches long, and 2 yards and 18 inches wide? 29 inches = 0.75 yds., 18 in. = 0.5 yds. $37.75 \times 2.5 = 94.37$ square yards.

Example 15. When butter costs 23 cents per pound, how much for 23 pounds? $23 \times 23 = \$5.29$.

Example 16. What will be the cost of a fly-wheel weighing 768 pounds, at 5 cents per pound? $768 \times 5 = \$38.40$.

When there are more than two factors to be multiplied together, consider the product of the two factors as one factor, and continue the multiplication with the next factor as before described. It matters not how many factors there may be, the multiplication can be continued to any extent, but a correct account of the index of the factors must be kept, as described on page 8. All the products which come between the operations need not be observed, only the last product.

A Calculating Machine of only nine inches in diameter, can not give more than the four first figures correct; the rest must be filled up with ciphers (0). In common calculations, a result in four figures will generally suffice. For higher numbers, the remainder of the figures are commonly ciphers; these the Machine tells to any extent. But if the calculation is an important one, requiring a dozen or more figures in the result, make the calculation by hand, but do not trust to the first result being correct. The second time prove the calculation by the machine; by it you are sure of obtaining the four first figures correct, which are always most important; for the last figures the operation by hand can be trusted.

We will see by the following, that this instrument easily manœuvres numbers with 10 or 20 figures, but in such examples there is more or less divisions or extraction of roots, which considerably reduces the number of figures before the result is obtained.

Example 17. $2 \times 3 \times 4 = 24$. Set the arm A on 2, B on Zero, fasten C, move the arms until B comes to 3, fasten A^e, loose C, set B on Zero, fasten C, loose A^e, move the arms until B comes to 4; the arm A will show the product = 24.

It will be seen in this example, that the product of 2 and 3 was not noticed; the multiplication was continued by 4, and the product first noticed was 24.

Example 18. What will be the cost of a lot of ground 135 feet long and 73 feet wide, at 6 cents per square foot? $135 \times 73 \times 0.06 = \$591.30$. Set A on 135, B on Zero, fasten C, move the arms until B comes to 73, fasten A^e, loose C, set B on Zero, fasten C, loose A^e, move the arms until B comes to 6; then the arm A will show the price = \$591.30.

Example 19. What will be the cost of a piece of cloth, 63 yards long and $2\frac{1}{2}$ yards wide, at \$1.56 per square yard? $63 \times 2.5 \times 1.56 = 236.25$.

Set A on 63, B on *Zero*, fasten C, move the arms until B comes to 2·5, fasten A^e, loose C, set B on *Zero*, fasten C, loose A^e, move the arms until B comes to 1·56; the arm A will show the product = \$236·25.

Example 20. What is the area of a circle 7 inches in diameter? $7 \times 7 \times 0\cdot785 = 38\cdot48$. Set A on 7, B on *Zero*, fasten C, move the arms until B comes to 7, fasten A^e, loose C, set B on *Zero*, fasten C, loose A^e, move the arms until B comes to 0·785; then B shows the area = 38·48 square inches.

Example 21. What is the area of a circle with a radius of 11 feet? $11 \times 11 \times 3\cdot14 = 95$ square feet.

When a number is to be multiplied by itself, it will be marked with an exponent of the power, as $11 \times 11 = 11^2$, $12 \times 12 \times 12 = 12^3$.

Example 22. How many cubic inches are contained in an iron bar 3 inches square and 3 feet long? $3^2 \times 12 \times 3 = 324$.

Example 23. How many cubic inches are contained in an iron bar 27 inches long, 3 inches wide, and 2 inches thick? $27 \times 3 \times 2 = 162$ cubic inches.

Example 24. How many cubic inches are contained in an iron bar 2 inches in diameter and 5 feet long? $2^2 \times 0\cdot785 \times 12 \times 5 = 188\cdot4$.

Example 25. What will be the weight of an iron bar 3 inches in diameter and 7 feet 6 inches long, if one cubic inch weighs 0·28 pounds? $3^2 \times 0\cdot785 \times 12 \times 7\cdot5 \times 0\cdot28 = 178$ pounds.

Example 26. What will be the price of an iron bar 9 feet 3 inches long, $3\frac{1}{2}$ inches wide, and $\frac{3}{4}$ inches thick, at 7 cents per pound? $9\cdot25 \times 3\cdot5 \times 0\cdot75 \times 0\cdot28 \times 7 = \$0\cdot47\cdot5$.

Example 27. What will be the weight of a propeller shaft of wrought iron 61 feet long and 1 foot in diameter, when a cubic foot of wrought iron weighs 486·6 pounds. $1^2 \times 0\cdot785 \times 61 \times 486\cdot6 = 23,300$ pounds.

DIVISION.

Division is the reverse of Multiplication, as the dividend is the product of the divisor and quotient.

RULE II. Set the arm A on the dividend, and B on the divisor, (circle a,) fasten C, move the arms until B comes to *Zero*; then the arm A shows the quotient.

Example 1. Divide 6 by 2. Set A on 6, B on 2, fasten C, move the arms until B comes to *Zero*; then A shows the product = 3.

Example 2. Divide 8 by 4. Set A on 8, B on 4, fasten C, move the arms until B comes to *Zero*; then A shows the product = 2.

Example 3. Divide 12 by 4. Set A on 12, B on 4, fasten C, move the arms until B comes to *Zero*; then A shows the product = 3.

Example 4. Divide 24 by 6. $24 : 6 = 4.$

" 5. " 36 " 4. $36 : 4 = 9.$

" 6. " 48 " 6. $48 : 6 = 8.$

" 7. " 63 " 9. $63 : 9 = 7.$

" 8. " 72 " 9. $72 : 9 = 8.$

" 9. " 125 " 5. $125 : 5 = 25.$

" 10. " 266 " 7. $266 : 7 = 38.$

" 11. " 378 " 9. $378 : 9 = 42.$

" 12. " 864 " 27. $864 : 27 = 32.$

" 13. " 5032 " 68. $5032 : 68 = 74.$

Example 14. A piece of ribbon containing 12 yards, was sold for \$3. What was the price per yard? Set A on 3, B on 12, fasten C, move the arms until B comes to *Zero*; then A shows the answer = 00.25 cents.

Example 15. If 23 yards of cloth cost \$161, what will be the price per yard? Set A on 161, B on 23, fasten C, move the arms until B comes to *Zero*; A will show the answer = \$7.

Example 16. If a cog wheel weighing 4750 pounds cost \$190, what will be the price per pound? Set A on 190, B on 4750, fasten C, move the arms until B comes to *Zero*; A will show the answer = 4 cents.

Example 17. If a screw propeller of cast iron, weighing 6950 pounds, cost \$417, what will be the price per pound? $417 : 6950 = 0.06$ cents.

When the dividend contains more than one factor, multiply the factors by Rule I., and divide by the divisor according to Rule II.

Example 18. If 27 men can do a certain amount of work in 7 days, how many days will it take for 21 men to do the same work?

$$27 : 21 = x : 7. \quad x = \frac{27 \times 7}{21} = 9.$$

Set A on 27, B on *Zero*, fasten C, move the arms until B comes to 7, fasten A^e, loose C, set B on 27, fasten C, loose A^e, move the arms until B comes to *Zero*; then the arm A shows the result = 9 days.

Example 19. What is the area of a triangle 7 feet in height, the base being 12 feet?

$$\text{Area} = \frac{7 \times 12}{2} = 42 \text{ square feet.}$$

Set A on 7, B on *Zero*, fasten C, move the arms until B comes to 12, fasten A^e, loose C, set B on 2, fasten C, loose A^e, move the arms until B comes to *Zero*; then A shows the result = 42 square feet.

Example 20. How many revolutions does the driving wheel of a locomotive make between Philadelphia and Baltimore, the distance being 99 miles, and the circumference of the wheel 27 feet?

$$\frac{99 \times 5280}{27} = 19,360 \text{ revolutions.}$$

This example can be computed in a single operation, as follows: Set A on 99, B on 27, fasten C, move the arms until B comes to 5280; then A shows the result = 19,360.

Example 21. What is the distance between Baltimore and Washington, a locomotive's driving wheel of 18 feet circumference making 11,733 revolutions on the road?

$$\frac{18 \times 11,733}{5280} = 40 \text{ miles.}$$

Set A on 18, B on 5280, fasten C, move the arms until B comes to 11733; then A shows 40 miles.

Example 22. How many cubic inches are there in a sphere 4 inches in diameter?

$$\frac{\pi d^3}{6} = \frac{3.14 \times 4^3}{6} = 33.5 \text{ cubic inches.}$$

Set A on 3.14, B on 6, fasten C, move the arms until B comes to 4, fasten A^e, loose C, set B on Zero, fasten C, loose A^e, move the arms until B comes to 4; then A shows the result = 33.5.

When the divisor contains more than one factor, divide by the first factor, as aforesaid, consider the quotient as a new dividend, and continue the division by the next factor.

Example 23. Divide 348 by 3 and 4.

$$\frac{348}{3 \times 4} = 29.$$

Set A on 348, B on 3, fasten C, move the arms until B comes to Zero, fasten A^e, loose C, set B on 4, fasten C, loose A^e, move the arms until B comes to Zero; then A shows the quotient = 29.

Example 24. How many days will it take for a steamboat to run from New York to Liverpool, running 12 miles per hour, the distance being 3100 miles?

$$\frac{3100}{12 \times 24} = 10.76.$$

Set A on 3100, B on 12, fasten C, move the arms until B comes to Zero, fasten A^e, loose C, set B on 24, fasten C, move the arms until B comes to Zero; then A shows = 10.74. Set A on 74, B on Zero, fasten C, move the arms until B comes to 24, A shows 17.76; then the answer is 10 days 17 $\frac{3}{4}$ hours.

When there is more than one factor in both the dividend and divisor, perform the operation as follows:—

$$\text{Example 25. } \frac{27}{436} \int \frac{36}{18} \int \frac{342}{21} \int \frac{79}{\text{---}} = 159 \cdot 8.$$

In this example, it will be necessary to keep a correct account of the factor's index, which will be as follows: Set A on 27, B on 436, fasten C, move the arms until B comes to 36, (the index for 27 and 36 is 2, the index for 436 = -2; that is, +2 - 2 = 0; set the hand on the screw C on 0,) fasten A^e, loose C, set B on 18, fasten C, loose A^e, move the arms until B comes to 342, fasten A^e, (the index for 342 = +2, index 18 = -1; that is +2 - 1 = +1, which is to be added to the former index = 0 + 1 = 1; set the small hand on +1,) loose C, set B on 21, fasten C, loose A^e, move the arms until B comes to 79, fasten A^e, (the index 21 = +1, index 79 = -1; but when the arm A moves over *Zero* in the direction of the arrow, 1 must be added to the index, as the sum of the factor's mantissa is more than 1, as in this last operation; set the small hand on +2, which shows that there must be three figures in the result;) then A shows the result = 159·8.

This result of seven factors was obtained by three operations, which, by the ordinary method of calculation, would be six operations. The account of the index is but trifling after a little practice. If there is a supposition as to what the number of figures in the result will be, the account of the index need not be kept.

Example 26. A ship's crew of 300 men were so supplied with provisions for 12 months, that each man was allowed 30 ounces per day, but after being out 6 months, they found it would require 9 months more to finish it, and 50 of their number were lost. It is required to find the daily allowance of each man during the last nine months.

$$\frac{300 \times 30 (12 - 6)}{9 (300 - 50)} = \frac{300}{9} \int \frac{30}{250} \int \frac{6}{\text{---}} = 24 \text{ ounces.}$$

Set A on 300, B on 9, fasten C, move the arms until B comes to 30, fasten A^e, loose C, set B on 250, fasten C, loose A^e, move the arms until B comes to 6; then A shows the result = 24 ounces. Here is five factors computed in two operations.

Example 27. If 288 men, in 5 days, working 11 hours a day, can dig a trench 220 yards long, 3 yards wide, and 2 yards deep, in how many days, working 9 hours a day, will 24 men dig a trench 240 yards long, 7 yards wide, and 3 yards deep?

$$\frac{288}{220} \int \frac{5}{3} \int \frac{11}{2} \int \frac{240}{24} \int \frac{7}{9} \int \frac{3}{\text{---}} = 230 \text{ days.}$$

Set A on 288, B on 220, fasten C, move the arms until B comes to 5, fasten A^e, loose C, set B on 3, fasten C, loose A^e, move the arms until

B comes to 11, fasten A^e , loose C, set B on 2, fasten C, loose A^e , move the arms until B comes to 240, fasten A^e , loose C, set B on 24, fasten C, loose A^e , move the arms until B comes to 7, fasten A^e , loose C, set B on 9, fasten C, loose A^e , move the arms until B comes to 3; then A shows the answer = 280 days.

Practice this example by keeping account of the indices.

PROPORTION.

RULE III. $a : b = c : d$. Set the arm A on the first term a , and B on the second term b , fasten C, move the arms until the arm A comes to the third term c ; then the arm B shows the fourth term d . Or, if the third term is to be found, set the arm B on the fourth term; then the arm A shows the third term; or in any position the arms may set, (when screwed together,) the number within them will remain in the same proportion as $a : b$.

Example 1. $2 : 3 = 4 : 6$. Set A on 2, B on 3, fasten C, move the arms until A comes to 4; then B shows the fourth term 6.

Example 2. $8 : 14 = 24 : x$. Set A on 8, B on 14, fasten C, move the arms until A comes to 24; then B shows $x = 42$.

Example 3. What is the interest of 568 dollars at 5 per cent. per annum? $100 : 5 = 568 : x$. Set A on 100, (*Zero*), B on 5, fasten C, move the arms until A comes to 568; then the arm B shows the interest = \$28.40.

Example 4. If three pounds of butter cost 87 cents, what will be the price of 8 pounds? $3 : 87 = 8 : x$. Set A on 3, B on 87, fasten C, move the arms until A comes to 8; then B shows the answer = \$2.52.

See Miscellaneous Examples.

FRACTIONS.

To reduce a vulgar fraction to a decimal, or to another vulgar fraction.

RULE IV. Set the arm A on the numerator, and B on the denominator, fasten the arms with C, move the arms until B comes to *Zero*; then A shows the decimal.

Example 1. Reduce $\frac{1}{2}$ to a decimal. Set A on 1, B on 2, fasten C, move the arms until B comes to *Zero*; then A shows the decimal = 0.5.

Example 2. Reduce $\frac{3}{4}$ to a decimal. Set A on 3, B on 4, fasten C, move the arms until B comes to *Zero*; then A shows the decimal = 0.75.

Example 3. Reduce $\frac{5}{9}$ to a decimal. Set A on 5, B on 9, fasten C, move the arms until B comes to *Zero*; then A shows the decimal = 0.5555.

<i>Example 4.</i>	Reduce	$\frac{7}{13}$	.	.	.	= 0.5384.
" 5.	"	$\frac{9}{15}$	.	.	.	= 0.6000.
" 6.	"	$\frac{12}{19}$	.	.	.	= 0.6368.
" 7.	"	$\frac{16}{27}$	.	.	.	= 0.5925.
" 8.	"	$\frac{38}{79}$	.	.	.	= 0.4810.
" 9.	"	$\frac{126}{372}$	.	.	.	= 0.3656.
" 10.	"	$\frac{793}{5934}$	.	.	.	= 0.1338.

When the denominator is more than ten times, or between ten and a hundred times, the numerator, the first figure of the decimal will express the hundredth part, and will be observed as follows:

Example 11. Reduce $\frac{3}{539}$ to a decimal. Set A on 3, B on 539, fasten C, move the arms until B comes to *Zero*; then A shows the decimal = 0.05566; for 3 the index = 0, and for 539 the index = 2; that is, 0 — 2 = —2, which shows there should be two ciphers before the figures, including the unity cipher.

<i>Example 12.</i>	Reduce	$\frac{7}{269}$	.	.	.	= 0.02602.
" 13.	"	$\frac{11}{2500}$	.	.	.	= 0.00440.
" 14.	"	$\frac{27}{369000}$	.	.	.	= 0.0007317.
" 15.	"	$\frac{33}{7773000}$	.	.	.	= 0.00004243.

In these examples it must be observed, that when the arm A moves over *Zero*, opposite the arrow, one must be subtracted from the index; when the arms are moved with the arrow, and A *not* moved over *Zero*, it is the same as if the arm A was moved over *Zero* with the arrow, and it is the same as when A is not moved over *Zero* opposite the arrow.

If it is made a rule to move the arms *with* the arrow for multiplication, and *opposite* the arrow for division, it follows that when, for multiplication, the arm A moves over *Zero*, we add 1 (+1) to the index, and for division subtract 1 (—1) from the index; but as it is more convenient to move the arm to the nearest factor, these rules will not be followed, and the arms will be moved in every direction for any operation.

Example 16. Reduce $\frac{7}{8}$ to a decimal. Set the arm A on 7, B on 8, fasten C, move the arms until B comes to *Zero*; then A shows = 0.875. In this example, as a division the arms should be moved opposite the arrow, but as the arm B is nearest *Zero with* the arrow, we prefer moving the arms in that direction, as the result will be precisely the same.

To reduce a vulgar fraction to another vulgar fraction.

Example 17. Reduce $\frac{3}{4}$ to 12ths. Set the arm A on 3, B on 4, fasten C, move the arms until B comes to 12; then A shows 9 or $\frac{9}{12} = \frac{3}{4}$. In whatever position the arms are moved on the disk, the numbers will

always remain in the same proportion as 3 : 4. In this consists the great convenience of the machine for working vulgar fractions.

Example 18. Reduce $\frac{1}{2}\frac{3}{1}\frac{5}{6}$ to 8ths. Set A on 135, B on 216, fasten C, move the arms until B comes to 8; then A shows =5 or $\frac{5}{8}$.

Example 19. Reduce $\frac{1}{3}\frac{8}{7}$ to 3ds. = $1\frac{4}{3}$.

To multiply a whole number by a vulgar fraction.

Example 20. Multiply 24 by $\frac{3}{4}$. $\frac{3}{4}\times 24=18$. Set A on 3, B on 4, fasten C, move the arms until B comes to 24, then A shows the product =18.

Example 21. $\frac{1}{3}\frac{9}{9}\times 365=180\cdot 4$.

Example 22. $\frac{2}{2}\frac{3}{2}\frac{3}{2}\times\frac{3}{3}\frac{7}{9}=0\cdot 04915$.

INVOLUTION.

The power of a number is only a multiplication of the number itself, and is marked by a small figure called an exponent, on the right of the number; as, if the letter a represents the number,—

$a^0=1$, no power of a .

a^1 =the first power of a .

a^2 =the second power or square of a .

a^3 =the third power or cube of a .

a^4 =the fourth power or bisquare of a .

Example 1. What is the square of 3? $3^2=9$. Set A on 3, B on *Zero*, fasten C, move the arms until B comes to 3; then A shows the power =9. To obtain the third power of 3, continue the multiplication by 3; but the fourth power of 3, or

$$3^4=9^2=81.$$

$$3^6=9^3=729.$$

$$3^8=9^4=81^2=6561.$$

$$3^9=9^4\times 3=81^2\times 36561\times 3=19683.$$

Example 2. Compute the 9th power of 3. Set A on 3, B on *Zero*, fasten C, move the arms until B comes to 3; then A shows $9=3^2$; fasten A^e, loose C, set B on *Zero*, fasten C, loose A^e, move the arms until B comes to 9; then A shows $81=9^2=3^4$; fasten A^e, loose C, set B on *Zero*, fasten C, loose A^e, move the arms until B comes to 81; then A shows $6561=81^2=9^4=3^8$; fasten A^e, loose C, set B on *Zero*, fasten C, loose A^e, move the arms until B comes to 3; then A shows the 9th power of 3, or $19683=3^9$.

Example 3. What is the 24th power of $1\frac{1}{2}$? Set A on 1·5, B on *Zero*, fasten C, move the arms until B comes to 1·5, fasten A^e; then A shows

$2 \cdot 25 = 1 \cdot 5^2$; loose C, set B on *Zero*, fasten C, loose A^e, move the arms until B comes to $2 \cdot 25$, fasten A^e; then A shows $5 \cdot 0625 = 1 \cdot 5^4$; loose C, set B on *Zero*, fasten C, loose A^e, move the arms until B comes to $2 \cdot 25$, fasten A^e; then A shows $11 \cdot 39 = 1 \frac{1}{2}^6$; loose C, set B on *Zero*, fasten C, loose A^e, move the arms until B comes to $11 \cdot 99$, fasten A^e; then A shows $129 \cdot 73 = 1 \cdot 5^{12}$; loose C, set B on *Zero*, fasten C, loose A^e, move the arms until B comes to $129 \cdot 73$, fasten A^e; then A shows $16830 = 1 \cdot 5^{24}$.

EVOLUTION.

Extraction of Roots is the reverse of Powers; as, the root of a number taken to the same power as the index for the root is coequal to the number. (See page 7.) This is to be computed by the circle *b*, which contains the logarithms for the number in circle *a*.

Example 1. What is the square root of 4, or $\sqrt{4}$? Set the arm A on 4, which you will find on circle *b*; the arm A shows 608, which is the mantissa of 4, and is to be divided by $2 = \text{index of the root}$. $608 : 2 = 301$. Say 2 in 6 three times; then set the arm B on 3 (circle *b*); 2 in 0 is 0, and 2 in 2 is 1; move the arm B until the curved line 3 crosses the first division on circle *b*, fasten B^e; then you will find on circle *a* the answer $= 2 = \sqrt{4}$.

Example 2. What is the square root of 9, or $\sqrt{9}$? Set A on 9, the mantissa on circle *b* is 9542; say 2 in 9 four times, set B on 4, (circle *b*), 2 in 15 7, move the arm until the curved line 4 crosses the division 7, (circle *b*), 2 in 14 seven times, move the arm B a little further, until the curved line crosses the seventh division between 7 and 8, counted from *b*; then fasten B^e, and you will find on circle *a* the answer $= 3 = \sqrt{9}$.

Example 3. What is the square root of 16, or $\sqrt{16}$? Set the arm A on 16, the mantissa on circle *b* is 2041, but 16 has an index $= 1$, which is to be added to mantissan, $1 \cdot 2041 : 2 = 0 \cdot 602$; set the arm B on 602, (circle *b*), on circle *a* we have the answer $= 4 = \sqrt{16}$.

Example 4. What is the cube root of 216, or $\sqrt[3]{216}$? Set A on 216, the mantissan on circle *b* is 3344, index $= 2$; $2 \cdot 3344 : 3 = 0 \cdot 7781$; set the arm B on 7781, (circle *b*), on circle *a* is the answer $= 6 = \sqrt[3]{216}$.

Example 5. What is the fourth root of $\sqrt[4]{69350}$? Set A on 69350, the mantissan on circle *b* is 841, index $= 4$; $4 \cdot 841 : 4 = 1 \cdot 2102$; set B on 3102, (circle *b*), the index for the answer is 1, and on circle *a* is the answer, $16 \cdot 23$.

Example 6. $\sqrt[7]{8926000}$. Set A on 8926000, the mantissan on *b* is 9506, index $= 6$; $6 \cdot 9506 : 7 = 0 \cdot 9929$, which corresponds with $9 \cdot 838$, the answer on circle *a*.

Example 7. What time will it take Jupiter to accomplish one revolution around the Sun, when we know that the square of the time for one revolution of two planets is in proportion as the cube of their distances?

The distance from the Sun to Jupiter is about 490,000,000 miles, and from the Sun to the Earth is about 95,000,000 miles. If we take the time for one revolution of the Earth around the Sun (one year) as a unit for measuring Jupiter's time, we have $x^2 : 1 = 490,000,000^3 : 95,000,000^3$.

$$\text{we have } x = \sqrt{\frac{490,000,000^3}{95,000,000^3}}.$$

Set A on 49, B on 95, fasten C, move the arms until B comes to 49, fasten A^e, loose C, set B on 59, fasten C, loose A^e, move the arms until B comes to 49, fasten A^e, loose C, set B on 95, fasten C, loose A^e, move the arms until B comes to Zero, fasten A^e; then A shows mantissan 1375, circle *b*, the index is 2; $2 \cdot 1375 : 2 = 1.0687$; set B on 0687, circle *b*, and you have on circle *a* the answer = 11.72 years. Set A on 72, B on Zero, fasten C, move the arms until B comes to 24, then A shows = 17.28 hours. Set A on 28, B on Zero, fasten C, move the arms until B comes to 60; then A shows 16.81 minutes. The time for Jupiter to accomplish a revolution around the Sun, is 11 years, 17 hours, 16.81 minutes.

When a root is to be extracted from a decimal fraction, it must be observed that the index for the mantissan is dependent on the root index.

Example 8. What is the distance from Venus to the Sun, when we know that Venus makes one revolution around the Sun in 225 days?

$$x^3 : 95,000,000^3 = 225^2 : 365^2;$$

$$\text{And } x = \sqrt[3]{\frac{95,000,000^3 \times 225^2}{365^2}} = 95,000,000 \sqrt[3]{\frac{225^2}{365^2}} =$$

Set A on 225, B on 365, fasten C, move the arms until B comes to 225, fasten A^e, loose C, set B on 365, fasten C, loose A^e, move the arms until B comes to Zero, fasten A^e; the index is —1, which is to be subtracted from the root index; the remainders $3 - 1 = 2$ is the index for the mantissan = 57978 on A, and $2 \cdot 5797 : 3 = 0.8599$. Set A on 8599, (circle *b*), fasten A^e, set B on Zero, fasten C, move the arms until B comes to 95; then A shows the distance = 68,810,000 miles.

Example 9. $\sqrt[3]{0.349}$. The given index is —1, which is to be subtracted from the root index = $3 - 1 = 2$, which will be the index for the mantissan. Set A on 349, fasten A^e; on circle *b* is the mantissa = 5428, and $2 \cdot 5428 : 3 = 0.8476$. Set A on 8476, circle *b*, then on the circle *a* will be found the root = 0.7041.

When the given index exceeds the root index, it will be as many ciphers

before the figures in the root as the root index contains in the given index, (excepting the unity cipher.)

Example 10. $\sqrt[4]{0\cdot0000000008945}$. The given index = -10 , and $10 : 4 = 2$; the remainder, 2, will be the index for the mantissa. Set A on 8945, the mantissan on circle *b* is 9515, and $2\cdot9515 : 4 = 0\cdot7379$. Set A on 7379, circle *b*; on circle *a* we have the root = $0\cdot005469$.

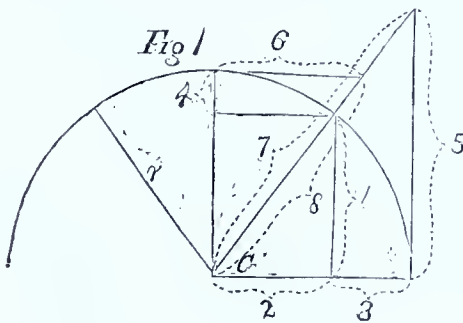
Example 11. $\sqrt[5]{0\cdot000000000007528}$. The given index = -13 , and $13 : 5 = 2$, which shows there must be two ciphers before the figures, (excepting the unity 0,) and the remaining 3 is the index for the mantissa. Set A on 7528, the mantissan on circle *b* is 8766, and $3\cdot8766 : 5 = 0\cdot7773$. Set A on 7733, circle *b*; on circle *a* we have the root = $0\cdot005989$.

TRIGONOMETRY.

Trigonometry is that part of Geometry which treats of triangles; it is divided into two parts, viz., *Plane* and *Spherical*. Plane Trigonometry treats of triangles which are (or are imagined to be) drawn on a plane; Spherical Trigonometry treats of the triangles which are (or are imagined to be) drawn on a sphere, so that the sides of the triangle are arcs of the great circle of the sphere.

A triangle contains seven quantities, namely, three sides, three angles, and the surface. When any three of these quantities are given, the four remaining ones can be ascertained by them, (one side or the area must be one of the given quantities,) and the operation is called solving the triangle, which is only an application of arithmetic on geometrical objects.

For the foundation of the above mentioned solution, there are assumed eight help quantities, which are called trigonometrical functions, and are the following: (see fig. 1.)



1. Sinus.
2. Cosinus.
3. Sinus-versus.
4. Cosinus-versus.
5. Tangent.
6. Cotangent.
7. Secant.
8. Cosecant.

r = Radius of the circle, which is

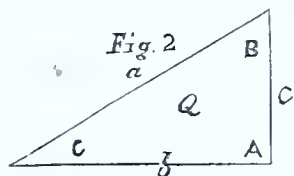
the unity by which the functions are measured.

These functions stand in a certain proportion to the sides and opposite

angles of the triangle; with their assistance the solution will be very simple.

In a triangle the sides are denoted by the letters a b c , their opposite angles by the letters A B C , and the area by Q . The sides a , b , and c bear the following relation to the trigonometrical functions:

Fig. 2, a right angled triangle.



$$a : c = r : \sin. C. \quad (1.)$$

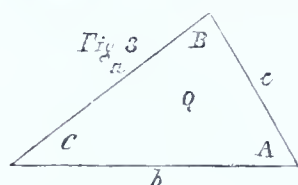
$$a : b = r : \cos. C. \quad (2.)$$

$$b : c = r : \text{tang. } C. \quad (3.)$$

$$b : a = r : \text{secant } C. \quad (4.)$$

$$\sin. C : \cos. C = \text{tang. } C : r. \quad (5.)$$

Fig. 3, an oblique angled triangle.



$$a : b = \sin. A : \sin. B. \quad (6.)$$

$$a : c = \sin. A : \sin. C. \quad (7.)$$

$$b : c = \sin. B : \sin. C. \quad (8.)$$

Tables of the trigonometrical functions are required for the common computation of these formulæ, but by the use of this simple instrument they are dispensed with, and it is so arranged that it is only necessary to notice the angles, not the functions.

Explanation.—The inner scale between c and d , marked on the arms, is for trigonometrical calculations. The numbers in the circle c and d represent the angles in degrees, the divisions between c and d show the exceeded minutes when the line crosses the arm. When the arm is set on an angle on circle c , the circle a shows the length of its sinus; when set on an angle on circle d , the circle a shows the length of its cosine; c' and d' show the complement for c and d .

Example 1. What is the length of $\sin. 30^\circ$? Set the arm A on the large 3, circle c ; the circle a shows the answer $=0.5$.

Example 2. What is the length of $\sin. 4^\circ 35'$? Set the arm A on the small 4, circle a , counted from *Zero*; move it a little further, until the fourth line crosses the 35th division on the arms, counted from c ; then on the arm, circle a shows the answer $=0.07991$.

Example 3. What is the length of $\cos. 63^\circ 42'$? Set the arm A on 63, circle d , move the arm a little further, until the 63d line crosses the 42d division on the arm, counted from d ; then circle a shows the length of $\cos. 63^\circ 42' = 0.44307$.

To find the Tangent and Cotangent.

From the analogy 5 we obtain—

$$\text{tang. } C = \frac{\sin. C}{\cos. C} \quad \text{when } r = 1.$$

Example 4. If the angle $C = 57^\circ 38'$, what will be the length of the tangent?

$$\text{Tang. } C = \frac{\sin. 57^\circ 38'}{\cos. 57^\circ 38'} = 1.578.$$

Set the arm A on $57^\circ 38'$, circle c , and the arm B on $57^\circ 38'$, circle d , fasten C, move the arm until B comes to *Zero*; then A shows the length of the tangent $C = 1.578$.

Example 5. What is the cotangent of $34^\circ 21'$?

$$\text{Cot. } C = \frac{\cos. 34^\circ 21'}{\sin. 34^\circ 21'} = 1.464.$$

Set A on $\cos. 34^\circ 21'$, circle d , and B on $\sin. 34^\circ 21'$, fasten C, move the arms until B comes to *Zero*; then A shows the length of the cotangent $34^\circ 21' = 1.464$.

To find the Secant and Cosecant.

Example 6. What is the length of the secant for an angle of $74^\circ 18'$?

$$\text{Secant } C = \frac{1}{\cos. 74^\circ 18'} = 3.695.$$

Set A on *Zero*, B on $\cos. 74^\circ 18'$, circle d , fasten C, move the arms until B comes to *Zero*; then A shows the length of the secant $74^\circ 18' = 3.695$.

Example 7. If the angle C be $48^\circ 7'$, what will the cosecant be?

$$\text{Cosecant } C = \frac{1}{\sin. 48^\circ 7'} = 1.343.$$

Set A on *Zero*, B on $\sin. 78^\circ 7'$, circle c , fasten C, move the arms until B comes to *Zero*; then A shows the length of cosecant $78^\circ 7' = 1.343$.

Example 8. In fig. 2, we have $a = 2360$ yards, and $C = 34^\circ 21'$; what will be the length of the side C ? From the analogy 1 we obtain—

$$c = a. \sin. C = 2360 \times \sin. 34^\circ 20' = 1331.$$

Set A on 2360, B on *Zero*, fasten C, move the arms until B comes to $\sin. 34^\circ 20'$, circle c ; then A shows the length of $c = 1331$ yards.

Example 9. An inclined plane is to be built for a railroad, the highest point of which is to be 264.5 feet above the base, and to form an angle with the horizon of not more than $10^\circ 25'$. What will be the length of the inclined plane?

$$\text{Length} = \frac{264.5}{\sin. 10^\circ 25'} = 1463 \text{ feet.}$$

Set A on 264.5, B on $\sin. 10^\circ 2'$, circle c , fasten C, move the arms until B comes to *Zero*; then A shows the length $= 1463$.

Example 10. In fig. 3 we have $c = 2354$ yds., $A = 32^\circ 20'$, $C = 50^\circ 34'$. What will be the length of the lines a and b ?

From the 7th analogy we obtain—

$$a = \frac{c \sin. A}{\sin. C} = \frac{2354 \times \sin. 32^\circ 20'}{\sin. 50^\circ 34'} = 1630.$$

Set A on 2354, B on $\sin. 50^\circ 34'$, fasten C, move the arms until B comes to $\sin. 32^\circ 20'$; then A shows the length of the side $c=1630$.

$$B=100 - A - B = 180 - 32^\circ 20' - 50^\circ 34' = 97^\circ 6'.$$

From the 8th analogy we obtain—

$$b = \frac{c \sin. B}{\sin. C} = \frac{2354 \times \sin. 97^\circ 6'}{\sin. 50^\circ 34'} = 3024.$$

Example 11. What will be the distance between an hostile intrenchment and two batteries, A and B, the distance between the two batteries being 1530 yards, the aiming angle at A $69^\circ 54'$, and at B $85^\circ 26'$?

$$\text{The distance from A} = \frac{1530 \times \sin. 85^\circ 26'}{\sin. 24^\circ 40'} = 3654.$$

$$\text{From B the distance is B} = \frac{1530 \times \sin. 69^\circ 54'}{\sin. 24^\circ 40'} = 3443.$$

Example 12. What will be the length of the side a , fig. 3, when $b=539$ yards, and $c=693$ yards? $A=41^\circ 37'$.

$$a = \sqrt{b^2 + c^2 - 2bc \cos. A}.$$

$$a = \sqrt{539^2 + 693^2 - 2 \times 539 \times 692 \times \cos. 41^\circ 37'} = 466 \text{ yards.}$$

Compute the square by the machine, and set it up as follows:—

$$\begin{array}{rcl} 539^2 & = & 290521 \\ 693^2 & = & 480249 \\ \hline & & 770770 \end{array}$$

According to Rule I, and cosine on circle d , multiply $2 \times 539 \times 693 \times \cos. 41^\circ 37' = 558500$.

$$\begin{array}{rcl} \text{From} & . & . & . & . & 770770 \\ \text{Subtract} & . & . & . & . & 558500 \\ \hline & & & & & 212270 \end{array}$$

$$\text{Set the arm A on} \quad . \quad . \quad 212270$$

The mantissa will be found on circle b , 32675, and the index $=5$; or, $5.32675 : 2 = 2.66337$, circle b , and circle a shows the answer $a=466$ yards.

Example 13. In a triangle, fig. 3, the side $b=356$ feet 3 inches, and $c=257$ feet 9 inches; the angle $A=25^\circ 13'$. What will be the area of the triangle?

$$Q = \frac{b c \sin. A}{2} = \frac{356.25 \times 257.75 \times \sin. 25^\circ 13'}{2} = 19560.$$

Example 14. What will be the length of the side c in the triangle, fig. 3, when the area is $Q=12677000$ square feet, and the angles $A=48^\circ 55'$, $C=56^\circ 30'$?

$$A + C = 105^\circ 25'.$$

$$c = \sqrt{\frac{2 Q \sin. C}{\sin. A \sin. (A \times C)}} = \sqrt{\frac{2 \times 12677000 \times \sin. 56^\circ 30'}{\sin. 45^\circ 55' \times \sin. 105^\circ 55'}} = 5533.$$

Set A on 2, B on sin. $48^\circ 55'$, circle c , fasten C, move the arms until B comes to 12677000, fasten A^e , loose C, set B on sin. $105^\circ 55'$, circle c , fasten C, loose A^e , move the arms until B comes to sin. $76^\circ 30'$, circle c , fasten A^e , the index is 7, the mantissan = 46374; $7 \cdot 46374 : 2 = 3 \cdot 73187$; set B on 73187, circle b ; the circle a shows the length of the side $c = 5533$.

Example 15. What will the angle C be when the sides $a = 5568 \cdot 6$ feet, and $c = 3247$ feet? (fig. 2.)

$$\sin. C = \frac{c}{a} = \frac{3247}{5568 \cdot 6} = 35^\circ 40', \text{ (circle } c.)$$

Example 16. What will the angle C be when the sides $a = 7804400$ yards, and $c = 4391589$ yards; $A = 125^\circ 2 \cdot 16'$?

$$\sin. C = \frac{c \sin. A}{a} = \frac{4391589 \times \sin. 125^\circ 2 \cdot 16'}{7804400} = 27^\circ 26 \cdot 3'.$$

Set A on 4391589, B on 7804400, fasten C, move the arms until B comes to sin. $125^\circ 2 \cdot 16'$, circle c ; then A shows on circle c the angle $C = 27^\circ 26 \cdot 3'$.

Example 17. Two ships of war notice a strong firing from a castle; in order to be safe, they keep themselves at a distance beyond the reach of the balls from the castle. To measure the distance from the castle, they place the vessels 880 yards from each other, and observe the angles between the castle and the vessels to be $A = 63^\circ 45'$, $B = 75^\circ 50'$. What will be the two distances from the castle?

$$C = 180 - 63^\circ 45' - 75^\circ 50' = 40^\circ 25'.$$

To A the distance will be—

$$b = \frac{c \sin. B}{\sin. C} = \frac{880 \times \sin. 75^\circ 50'}{\sin. 40^\circ 25'} = 1316.$$

Set A on 880, B on sin. $40^\circ 25'$, circle c , fasten C, move the arms until B comes to sin. $75^\circ 50'$, circle c ; then A shows the distance = 1316.

$$a = \frac{c \sin. A}{\sin. C} = \frac{880 \times \sin. 63^\circ 45'}{\sin. 40^\circ 25'} = 1218.$$

Example 18. From a window in the lower floor of a house which lays level with the foot of a tower, is observed an angle to the top of the tower, $A = 40^\circ$ from the horizon; from another window in the upper story, in the same perpendicular as the lower window, the altitude of the tower is observed to be $C = 37^\circ 30'$, which is 18 feet above the lower window.

Then we have $A = 90 - 40^\circ = 50^\circ$. $C = 90 + 37^\circ 30' = 127^\circ 30'$.

$$B = 180 - 50 - 127^\circ 39' = 2^\circ 30'. \quad b = 18 \text{ feet.}$$

What will be the height of the tower and the distance from it?

The distance from the lower window to the top of the tower = c .

$$c = \frac{b \sin. C'}{\sin. B} = \frac{18 \times \sin. 127^\circ 30'}{\sin. 2^\circ 30'} = 327.3 \text{ feet.}$$

The height of the tower = h .

$$h = c. \sin. A = 327.3 \times \sin. 40^\circ = 210 \text{ feet.}$$

The distance to the tower = d .

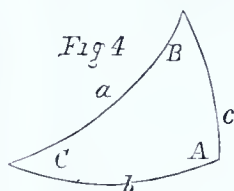
$$d = c. \cos. A = 327.3 \times \cos. 40^\circ = 250.8 \text{ feet.}$$

SPHERICAL TRIGONOMETRY.

Spherical Trigonometry treats of the triangles which are, or are imagined to be, drawn on the surface of a sphere, the sides of which are arcs of the great circle of the sphere, and measured by the angle of the two radii drawn from the centre to the ends of the arcs.

In a spherical triangle, the trigonometrical functions bear quite a different relation to the sides than in a plane triangle, which will be seen by the following:—

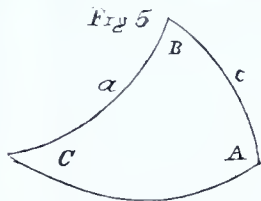
Fig. 4, A right angled spherical triangle.



A being the right angle.

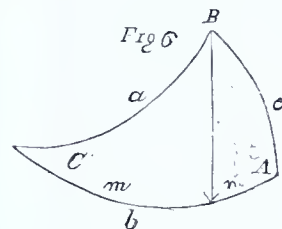
$$\begin{aligned} \sin. a : 1 &= \sin. c : \sin. C. \\ \text{tang. } a : 1 &= \text{tang. } b : \cos. C. \\ \cos. a : 1 &= \cot. B : \text{tang. } C. \\ \sin. c : 1 &= \text{tang. } b : \text{tang. } B. \end{aligned}$$

Fig. 5, An oblique angled triangle.



$$\begin{aligned} \sin. a : \sin. b &= \sin. A : \sin. B. \\ \sin. b : \sin. c &= \sin. B : \sin. C. \end{aligned}$$

Fig. 6. A line drawn from the angle B perpendicular to the side b , which two parts are denoted by the letters m and n .



$$\begin{aligned} \cos. a : \cot. m &= 1 : \text{tang. } C. \\ \text{tang. } a : \text{tang. } m &= 1 : \cos. C. \\ \cos. A : \text{tang. } c &= \sin. m : \sin. (b-m). \\ \cos. a : \cos. c &= \cos. m : \cos. (b-m). \\ \cos. B : \cos. m &= \text{tang. } a : \text{tang. } c. \end{aligned}$$

By Spherical Trigonometry we ascertain distance and angles on the surface of the earth. It is principally used in Navigation, and for that purpose, the above formula requires to be as simple as possible, and the quantities A, B, C , and a, b, c , expressed in courses, latitude and longitude.

In the above mentioned triangles, the sides are arcs of the great circle of the earth, or arcs of a circle plane drawn through the centre of the earth. When the latitude and longitude are given, we obtain in the difference of longitude circle planes which are drawn at a distance from the centre of the earth = sinus of the latitude, with a radius = cosine for latitude; then, in the parallel, the distance between two points in the same latitude will be obtained by multiplying the difference in longitude by the cosine for latitude, and the course will then be the same through all the distance, but this will not be the shortest distance between the two points, as the arc of a circle plane drawn through the centre of the earth, and then the vessel will always be in a higher latitude than the starting point. At half the distance, the vessel will be in the highest latitude, l' , which will be found by the formula—

$$\text{Tang. } l' = \frac{\sin. l}{\cos. \frac{1}{2} L \cos. l} \quad . \quad . \quad . \quad . \quad (1.)$$

In which the letters denote—

L = difference in longitude.

l = latitude at the starting point.

l' = highest latitude.

d = shortest distance in the arc of the great circle.

C = the course from the starting point.

$$\text{Sin. } \frac{1}{2} d = \sin. \frac{1}{2} L \cos. l. \quad . \quad . \quad . \quad . \quad (2.)$$

$$\text{Sin. } C = \frac{\sin. L \cos. l}{\sin. d}. \quad . \quad . \quad . \quad . \quad (3.)$$

Example 1. What will be the shortest distance between Boston, (North America,) and Cape Creaux, (South Spain,) which are in the same latitude,—about $42^\circ 20'$; and what will be the highest latitude and course from Boston?

Longitude of Boston west from Greenwich, $= 71^\circ 4'$

“ Cape Creaux east, $= 3^\circ 16'$

Difference in longitude, . . . $L = 74^\circ 20'$

Latitude of the two places, . . . $l = 42^\circ 20'$

By the insertion of these quantities in equation 1, we obtain the highest latitude:—

$$\text{Tang. } l' = \frac{\sin. 42^\circ 20'}{\cos. 37^\circ 10' \times \cos. 42^\circ 20'} = 48^\circ 50'.$$

Set the arm A on $\sin. 42^\circ 20'$, B on $\cos. 37^\circ 10'$, fasten C, move the arms until B comes to *Zero*, fasten A^e, loose C, set B on $\cos. 42^\circ 20'$, fasten C, loose A^e, move the arms until the arm A shows the same angle on circle c as the arm B shows on circle d ; it will be $48^\circ 50' =$ highest latitude.

By equation 2 we obtain the shortest distance:—

$$\text{Sin. } \frac{1}{2} d = \text{sin. } 37^{\circ} 10' \times \text{cos. } 42^{\circ} 20' = 26^{\circ} 32' \times 2 = 53^{\circ} 4'.$$

Set A on sin. $37^{\circ} 10'$, B on Zero, fasten C, move the arms until B comes to cos. $42^{\circ} 20'$; then A shows $26^{\circ} 32'$, circle c, which, multiplied by 2, will be $53^{\circ} 4' \times 60 = 3184$ miles.

The course from Boston by formula 3:—

$$\text{Sin. C} = \frac{\text{sin. } 74^{\circ} 20' \times \text{cos. } 42^{\circ} 20'}{\text{sin. } 53^{\circ} 4'} = 62^{\circ} 56'.$$

Set A on sin. $74^{\circ} 20'$, B on sin. $53^{\circ} 4'$, fasten C, move the arms until B comes to cos. $42^{\circ} 20'$; then A shows $62^{\circ} 56'$, which should be the course from the meridian, or about $5\frac{3}{4}$ points east.

When two places lay in different latitudes, and their longitudes are given, to find the nearest distance and course.

l = lower latitude.

l' = highest latitude.

C = course from latitude l .

C' = course from latitude l' .

d = shortest distance between l and l' .

L = difference in longitude.

$\tan. m = \cot. l', \cos. L$.

$n = 90 \pm l - m$.

— l , when l and l' are on one side of the equator.

+ l , when l is on one side and l' on the other.

$$\text{Cos. } d = \frac{\text{sin. } l' \cos. n}{\cos. m}. \quad (4.)$$

$$\text{Sin. C} = \frac{\text{sin. L} \cos. l'}{\cos. l}. \quad (5.)$$

$$\text{Sin. C}' = \frac{\text{sin. L} \cos. l}{\sin. d}. \quad (6.)$$

$$\text{Sin. C} = \frac{\text{sin. C}' \cos. l'}{\cos. l}. \quad (7.)$$

$$\text{Sin. C} = \frac{\text{sin. C} \cos. l}{\cos. l'}. \quad (8.)$$

Example 2. When the steamer *Atlantic* runs from New York to Liverpool in 9 days, what will be her course from New York, and shortest distance to Liverpool, and what will be her speed in nautical miles per hour?

$$\begin{aligned} l &= 40^\circ 42' \text{ N. latitude, } \left. \begin{array}{l} 74^\circ \text{ W. longitude,} \end{array} \right\} \text{New York.} \\ l' &= 53^\circ 22' \text{ N. latitude, } \left. \begin{array}{l} 2^\circ 52' \text{ W. longitude,} \end{array} \right\} \text{Liverpool.} \end{aligned}$$

$$L = 71^\circ 8' \text{ difference in longitude.}$$

$$\text{Tang. } m = \cot. 53^\circ 22' \times \cos. 71^\circ 8' = 13^\circ 32'.$$

Set the arm A on $\cos. 53^\circ 22'$, B on $\sin. 53^\circ 22'$, fasten C, move the arms until B comes to $\cos. 71^\circ 8'$, fasten A^e, loose C, set B on *Zero*, fasten C, loose A^e, move the arms until it shows the same angle on A, circle *c*, as on B, circle *d*; it will be found to be $13^\circ 32'$.

$$n = 90 - 40^\circ 42' - 13^\circ 32' = 35^\circ 46'.$$

$$(4.) \quad \text{Cos. } d = \frac{\sin. 53^\circ 22' \times \cos. 35^\circ 46'}{\cos. 13^\circ 32'} = 47^\circ 56'.$$

Set A on $\sin. 53^\circ 22'$, B on $\cos. 13^\circ 32'$, fasten C, move the arms until B comes to $\cos. 35^\circ 46'$; then A shows $d = 47^\circ 56'$, circle *d*.

$47.94 \times 60 = 2876$ miles, the shortest distance.

$$(5.) \quad \text{Sin. } C = \frac{\sin. 71^\circ 8' \times \cos. 53^\circ 22'}{\sin. 47^\circ 56'} = 49^\circ 37'.$$

$$(6.) \quad \text{Sin. } C' = \frac{\sin. 71^\circ 8' \times \cos. 40^\circ 42'}{\sin. 47^\circ 56'} = 104^\circ 54'.$$

$$C = \text{N. E. } \frac{1}{2} \text{ E.} \quad C' = \text{W. S. W. } \frac{3}{4} \text{ W.}$$

$$\text{Miles per hour} = \frac{47.833 \times 60}{24 \times 9} = 13.28.$$

Example 3. What will be the distance and course from San Francisco to Port Jackson, in New Holland?

$$\begin{aligned} l' &= 37^\circ 47' \text{ N. latitude, } \left. \begin{array}{l} 122^\circ 21' \text{ W. longitude,} \end{array} \right\} \text{San Francisco.} \end{aligned}$$

$$\begin{aligned} l &= 33^\circ 50' \text{ S. latitude, } \left. \begin{array}{l} 151^\circ 25' \text{ E. longitude,} \end{array} \right\} \text{Port Jackson.} \end{aligned}$$

$$L = 360 - 122^\circ 21' - 151^\circ 25' = 86^\circ 14'.$$

$$\text{Tang. } m = \cot. 37^\circ 47' \times \cos. 86^\circ 14' = 40^\circ 17'.$$

$$n = 90 + 33^\circ 50' - 40^\circ 17' = 83^\circ 33'.$$

$$(4.) \quad \text{Cos. } d = \frac{\sin. 37^\circ 47' \times \cos. 83^\circ 33'}{\cos. 40^\circ 17'} = 95^\circ 11'.$$

$95.18 \times 60 = 5711$ miles, the shortest distance.

$$(5.) \quad \text{Sin. } C = \frac{\sin. 86^\circ 14' \times \cos. 37^\circ 47'}{\sin. 95^\circ 11'} = 52^\circ 22'.$$

$$(6.) \quad \text{Sin. } C' = \frac{\sin. 86^\circ 14' \times \cos. 33^\circ 50'}{\sin. 95^\circ 11'} = 123^\circ 40'.$$

$$C = \text{N. E. } \frac{1}{2} \text{ E. from Port Jackson.}$$

$$C = \text{S. W. by W. from San Francisco.}$$

ADDITION AND SUBTRACTION.

Addition and Subtraction of numbers are to be computed on circle *b*. The large figures can represent the unity, or any multiple or division of ten. If we say the large figures to express the hundredths, the figures on the arm, counted from *b*, will express the tenths, and the divisions between those figures the unity.

Example 1. To set the arm A on 546. Set the arm A on the large 5, circle *b*, move it further, until the fifth curved line intersects the fourth (4) figure on the arm, counted from *b*, move it a little further, until the same curved line intersects the sixth (6) division between 4 and 5; then the arm A is set on 546.

Example 2. To 546
Add 368

914

Set A on 546, (as in *Ex. 1*.) B on Zero, fasten C, move the arms until B comes to 368; then the arm A shows the sum =914.

Example 3. From +895
Subtract -372

523

Set A on 895, B on 372, fasten C, move the arms until B comes to Zero; then A shows the difference =523.

Example 4. { +493
-355
+545

683

Set A on 493, B on 355, fasten C, move the arms until B comes to 545; then A shows the sum =683.

Example 5. { +763
-691
+244
-964
+865

217

Set A on 763, B on 691, fasten C, move the arms until B comes to 244, fasten A^e, loose C, set B on 964, fasten C, loose A^e, move the arms until B comes to 865; then A shows the amount =217.

In Addition, when the arm A moves over Zero, (with the arrow,) one must be carried to the thousands; for that purpose, use the small hand on

the screw C, which is to be operated by hand, as in Multiplication and Division. In Addition move the arms *with the arrow*, and for Subtraction *opposite the arrow*; then when the arm A moves over *Zero*, *add* one as a thousand, and when moved opposite the arrow, *subtract* one as a thousand.

$$\begin{array}{r} \text{Example 6.} \quad \left\{ \begin{array}{r} +795 \\ +843 \end{array} \right. \\ \hline 1638 \end{array}$$

Add this as before described, and it will be found that A moves over *Zero*, and shows 638, to which is to be added 1 as a thousand, or 1638.

In Subtraction, when the positive quantity is less than the negative, we subtract the positive from the negative; then the difference will be the negative. But if we subtract the negative from the positive, the difference will be positive; but then the arm A moves over *Zero*, opposite the arrow, and one is to be subtracted as a thousand.

$$\begin{array}{r} \text{Example 7.} \quad \begin{array}{r} +466 \\ -879 \end{array} \\ \hline 413 \end{array}$$

In this example, if we subtract 466 from 879, the difference will be —413, but if we subtract 879 from 466, the difference will be +587 —1000 = —413.

Example 8. A lady went shopping, and bought,—

1 pair of gloves,	\$0·87	
1 silk bonnet,	5·37	
And offered in payment,		\$10·00
The change will be		\$3·76

Set A on *Zero*, which then will represent \$10·00, set B on 0·87, fasten C, move the arms until B comes to *Zero*, fasten A^e, loose C, set B on 5·37, fasten C, loose A^e, move the arms until B comes to *Zero*; then A shows the change = \$3·76.

Example 9. In a hardware store, a bill of goods was sold to a carpenter, consisting of,—

1 axe,	\$1·37	
1 plane,	75	
1 bevel,	94	
And paid it with		\$5·00
The change will be		1·94

Set A on 5·00, B on 1·37, fasten C, move the arms until B comes to *Zero*, fasten A^e, loose C, set B on 0·75, fasten C, loose A^e, move the

arms until B comes to *Zero*, fasten A^e, loose C, set B on 0·74, fasten C, loose A^e, move the arms until B comes to *Zero*; then A shows the change = \$1·94.

Example 10. In a hardware store was bought the following articles:—

1 hammer,	\$0·50
1 anvil,	3·23
1 bench vise,	6·87
1 square,	0·75
1 set of figure punches,	4·33
Paid the bill with	\$20·00
The change will be	4·32

Set A on *Zero*, which, in this instance, will represent \$20, and operate as before described.

When the numbers which are to be added or subtracted contain more than three figures, the operation will be done in two or more divisions, by dividing the numbers into columns, so that three figures of each number will come in each column.

Example 11. {

+346,998
+436,845
+ 34,493
+ 6,936
+ 784
4,056
822
826056

Add the column of three figures at the right as before described, and set down the sum. Afterwards add the column at the left, and set down the sum as shown in the example.

Example 12. {

+368,815
—865,435
+585,936
— 46,938
+ 94,687
1,065
136
137,065

Example 13.

$$\begin{array}{r}
 +3,469,649,783 \\
 +8,364,573,206 \\
 -7,804,038,123 \\
 + 689,430,231 \\
 - 439,034,012 \\
 + 68,902,740 \\
 - 3,937,300 \\
 + 760,000 \\
 \hline
 344 \quad 1,525 \\
 4 \quad 2,305 \\
 \hline
 +4,346,306,525 \\
 \hline
 \end{array}$$

MISCELLANEOUS EXAMPLES.

The most convenient method of using the Calculating Machine, is to set up the calculation in the form of an equation.

Example 1. What will be the interest of \$4365 in 7 years at 6 per cent.?

If we first set it up as an analogy, as

$$\begin{array}{ccc}
 100 : 4365 = 6 : x \\
 1 \quad \quad 7
 \end{array}$$

Before commencing the operation with the machine, we set it up in the form of an equation, as—

$$x = \frac{4365 \times 7 \times 6}{100} = 1833.30.$$

Example 2. Eight persons, men and women, have spent 38 dollars; when the men spend 7 dollars the women spend four. How many of each sex were in the company?

$$\text{Men} = \frac{38 - (4 \times 8)}{7 - 4} = 2.$$

$$\text{Women} = \frac{(7 \times 8) - 38}{7 - 4} = 6.$$

Example 3. A locomotive starts from Philadelphia for Baltimore at 10 o'clock, and runs 20 miles per hour; another locomotive starts from Philadelphia on the same road at 10 $\frac{3}{4}$, but runs 25 miles per hour. The time and distance from Philadelphia is required, when the latter locomotive reaches the former?

The first locomotive is 15 miles ahead when the other one starts. Then the time will be—

$$\text{Time} = \frac{15}{25-20} = 3 \text{ hours.}$$

Distance $= 3 \times 20 = 60$ miles.

Example 4. A locomotive starts from Baltimore for Philadelphia at 8 o'clock, and runs $23\frac{3}{4}$ miles per hour; another locomotive starts from Philadelphia at 9 o'clock and 20 minutes, and runs 37 miles per $1\frac{1}{2}$ hours. It is required to know the time and distance from Philadelphia when they meet?

When the Philadelphia locomotive started, the Baltimore one had run $23 \times 5 \times 1.333 = 31.66$ miles, the distance from Baltimore being 99 miles, and $99 - 31.66 = 67.34$.

$$\text{Time} = \frac{67.34}{\frac{37}{1.5} + 23.75} = \frac{67.34}{48.42} = 1.391 \text{ hours.}$$

Distance from Philadelphia $\frac{1.391 \times 37}{1.5} = 34.3$ miles.

The time is $9.333 + 1.391 = 10.724 = 10$ o'clock and 43 minutes.

Example 5. What is the interest of \$987.87 for 18 months and 18 days, at 6 per cent. per annum?

$$\text{Interest} = \frac{987.78}{100} \int \frac{18.6}{12} \int^6 = 92.28.$$

Example 6. What is the amount of \$3687 for 2 years, 21 months, and 25 days, (45.866 months,) at 6 per cent. per annum?

$$\text{Amount} = 3687 + \frac{3687}{100} \int \frac{45.866}{12} \int^6 = \$4532.40.$$

Example 7. What capital, at interest for 9 years and 6 months, at $4\frac{1}{2}$ per cent. per annum, will amount to \$856.50?

$$\text{Capital} = \frac{100 \times 856.50}{100 + (4.5 \times 9.5)} = \$600.$$

Example 8. A father bequeathed to his son \$750, at 5 per cent. interest, to be received when he was 21 years of age; his guardian found the amount to be \$1096.85. How old was the boy at the time of his father's death?

$$\text{Age} = 21 - \frac{100(1096.85 - 750)}{750 \times 5} = 11.75 \text{ years.}$$

Example 9. A boy, 8 years of age, asked a mathematician, "How old are you, sir?" The mathematician replied, "If you multiply the difference between your age and mine, by my age, you will obtain the cube of your age." How old was the mathematician?

$$\text{Age} = \frac{8}{2} + \sqrt[3]{8^3 + \left(\frac{8}{2}\right)^2} = 27 \text{ years.}$$

Example 10. A house in London received a bill of exchange from Philadelphia for £943 17s. sterling. For how much was it drawn, exchange being \$3·8 per pound sterling?

$$17s. = 0·85£. \text{ Was drawn for } 943·85 \times 3·8 = \$3586·63.$$

PROGRESSION.

Example 11. How many strokes does a clock which strikes the hour make while the hour hand is going around once?

$$S = \frac{12(12+1)}{2} = 78 \text{ strokes.}$$

Example 12. A man who was engaged in putting down poles for an Electric Telegraph, had 125 poles collected in one place, to be set in a straight line 196 feet apart, and 165 feet from the collection was where the first one was to be placed. How many miles has he to travel when he sets the last one, and returns to the spot where the poles were first collected?

$$\frac{125}{5280} (2 \times 165 + 196 [125 - 1]) = 584·1.$$

Subtract 1 from 125, and multiply the remainder by 196, (Rule I;) to the product add 2×165 ; set the sum on A, and B on 5280, fasten C, move the arms until B comes to 125; then A shows the answer = 584·1 miles.

Example 13. Two steamboats leave New York at the same time, bound for Liverpool; one runs 10 miles per hour, the other runs only 2 miles the first hour, but each succeeding hour increases in speed $1\frac{1}{2}$ miles. It is required to know the time from New York, when the one which run 2 miles the first hour will pass the other?

$$\text{The time} = \frac{20 + 1·5 - 2 \times 2}{1·5} = \frac{17·5}{1·5} = 11·666.$$

That is, 11 hours 40 minutes; distance 116·6 miles.

Example 14. A gentleman bought an estate, which was to be paid for in 25 years. The first year he paid \$2000, and each following year he paid \$30 less than the first. What was the price of the property?

$$\text{Price} = 2000 \times 25 - \frac{30 \times 25}{2} (25 - 1) = \$41,000.$$

Example 15. A merchant has out \$37,900, with compound interest of 9 per cent. What will be the total sum in six years?

$$\text{Sum} = 37,900 (1·09)^6 = \$63,580.$$

Compute the 6th power of 1·09 as described in Involution, and multiply it by 37,900; the product is the answer = \$63,580.

Example 16. \$6,348,000 was to be paid after 15 years; by agreement it is paid immediately, deducting 7 per cent. compound interest from the principal. What will be the sum paid?

$$\frac{6,348,000}{(1.07)^{15}} = \$2,301,000, \text{ immediate payment.}$$

Example 17. \$794,600 is to be paid after 7 years and 4 months; but it is paid at once with \$304,500. With compound interest, what will be the per centage?

Per centage = p expressed in a fraction of 100.

$$\text{Log. } (1+p) = \frac{\text{log. } 79460 - \text{log. } 30450}{7.333} = 0.0568.$$

Set A on 7946, B on 3045, fasten C, move the arms until B comes to *Zero*; then A shows 4165, circle b . Set A on 4165, circle a , B on 7333, circle a , fasten C, move the arms until B comes to *Zero*; then A shows 0568, circle a . Set A on 0.0568, circle b , the corresponding number on circle a is 1.14, from which one is to be subtracted; the remainder is $p=14$ per cent.

Example 18. How many years will be required for a sum of money to double itself at 6 per cent. compound interest?

$$\text{Number of years} = \frac{\text{log. } 2}{\text{log. } 1.06} = \frac{0.30103}{0.02531} = 11.89 \text{ years.}$$

Set A on 2, circle a , the logarithm on circle $b=0.30103$; set A on 30103, circle a , fasten A^c, in the same manner set B on log. 1.06, circle a , fasten C, loose A^c, move the arms until B comes to *Zero*; then A shows on circle a the number of years = 11.89.

Reduce 0.89 to months and days as before described; the result will be 11 years, 10 months, 23.4 days.

Example 19. A yearly payment of \$65 has been received for 17 years. What is the amount of it with a simple interest of 6 per cent?

$$\text{Amount} = 65 \times 17 \left(1 + \frac{0.06}{2} [17 - 1] \right) = \$1635.40.$$

Example 20. What will be the amount, with compound interest at 5 per cent., of a yearly payment of \$43, received for 13 years?

$$\text{Amount} = \frac{43}{0.05} [(1 + 0.05)^{13} - 1] = \$761.10.$$

Example 21. A yearly payment of \$145 is paid in advance for 8 years. What will be the payment, deducting simple interest at 6 per cent.?

$$\text{Payment} = 145 \times 8 \left(1 - \frac{0.06 (8 - 1)}{2} \right) = \$916.40.$$

Example 22. In the preceding example, what will be the payment with compound interest?

$$\text{Payment} = \frac{145 \times 1.06^8 - 145}{0.06 \times 1.06^8} = \$900.30$$

Compute the eighth power as described on page 17.

Example 23. Two kinds of wine are to be mixed together to make 20 gallons, at \$2.25 per gallon; the one wine, y , cost \$1.87 per gallon, and the other, x , cost \$2.75;—5 per cent. is to be gained by the mixture. What will be the proportions of the wines?

$$x + y = 20.$$

$$2.25(x + y) = 1.05(2.75x + 1.87y).$$

$$x = \frac{20(2.25 - 1.05 \times 1.87)}{1.05(2.75 - 1.87)} = 6.2 \text{ gallons.}$$

$$y = 20 - 6.2 = 13.8 \text{ gallon.}$$

Example 24. A cask contains 90 gallons of wine, from which 1 gallon is taken every day, but is replaced by a gallon of water, so that the cask is always full of wine and water. How much water and how much wine is there in the cask after 8 days?

$$\text{After 8 days, there will be wine} = \frac{(90 - 1)^8}{90^{8-1}} = \frac{89^8}{90^7} = 81.8 \text{ galls.}$$

See Rule for Involution, page 17.

Take the eighth power of 89, the first figures will be found to be 3927; in the same manner take the eighth power of 90, and divide it by 90, the 7th power of 90 will be obtained, the first figures of which will be 4783.

$$\frac{3027}{4783} = 81.8 \text{ gallons.}$$

Example 25. Two alloys (a and b) of gold and silver are—

The one, $a = 20$ gold, 4 silver.

And the other, $b = 14$ “ 10 “

Specific gravity of pure gold, 19.25.

“ “ “ silver, 10.50.

From this alloy, there is to be obtained an alloy with 18 parts gold and 6 parts silver, weighing 30 ounces. What will be the proportion of the alloys a and b in ounces?

$$a = \frac{30 [19.25(18 - 14) - 10.5(10 - 6)]}{19.25(20 - 14) - 10.5(10 - 4)} = 20 \text{ ounces.}$$

$$b = 30 - a = 30 - 20 = 10 \text{ ounces.}$$

EXAMPLES FOR STEAM ENGINES.

Example 26. What is the power of a high pressure steam engine, with a cylinder = 27 inches diameter, and $s = 35$ inches stroke; the pressure

on the piston being $P = 60$ pounds per square inch, and making $n = 75$ single strokes per minute?

Allow $f = 20$ per cent. for friction.

$$H = \frac{a P s n}{33000} (1 - f) = \frac{27^2 \times 0.785 \times 60 \times 35 \times 75}{33000 \times 12} (1 - 0.20) = 182.66.$$

Example 27. What is the power of a condensing engine, the cylinder being $d = 38$ inches in diameter, stroke of piston $s = 4.5$ feet, pressure on piston $s = 11$ pounds per square inch, and vacuum $= 11$, $P = 22$ pounds, making $n = 30$ revolutions per minute?

$$H = \frac{a P s n}{33000} 2 (1 - f) = \frac{1134 \times 22 \times 4.5 \times 30}{33000} 2 (1 - 0.3) = 142.9.$$

EXAMPLES FOR A SCREW PROPELLER.

Example 28. What will be the pitch of a screw propeller 12 feet 9 inches in diameter $= D$?

When constructed for a direct action steam engine, a proper pitch is —

$$P = 2.5 D = 2.5 \times 12.75 = 31.875 \text{ feet.}$$

Example 29. What will be the slip of the propeller on a vessel with the greatest immersed sectional area?

$$Q^2 = 300 \text{ square feet.}$$

$$Q = 875 \text{ tonnage of displacement.}$$

$$A = 127.6 \text{ area of the propeller in square feet.}$$

$$S = \frac{Q + \sqrt[3]{Q^2}}{10 A} = \frac{300 + \sqrt[3]{875^2}}{10 \times 127.6} = 0.307.$$

or 30 per cent.

Example 30. What will be the horse power of a steam engine required to drive the same propeller $n = 50$ revolutions per minute?

$$H = \frac{A n^2}{1950} \sqrt{P S} = \frac{127.6 \times 50^2}{1950} \sqrt{32 \times 0.307} = 508 \text{ horses.}$$

Finish the root first, which will be 3.1; afterwards continue the operation with the other factors.

Example 31. What will be the speed of the vessel in statute miles per hour?

$$M = \frac{P n}{88} (1 - S) = \frac{31 \times 50}{88} (1 - 0.30) = 12.75 \text{ miles.}$$

Example 32. What will be the diameter of the cylinders for the above arrangement—two condensing engines; mean pressure on piston $P = 30$

pounds per square inch, diameter of cylinder $d=1.5$ s the stroke, friction and working the pumps being $f=0.30$?

$$d = \sqrt[3]{\frac{94 n A \sqrt{PS}}{P(1-f)}} = \sqrt[3]{\frac{94 \times 50 \times 127.6 \sqrt{32 \times 0.30}}{30(1-0.30)}} = 43.5 \text{ inches.}$$

$$s = \frac{d}{1.5} = \frac{43.5}{1.5} = 29 \text{ inches} = 2.415 \text{ feet.}$$

Begin the operation on $\sqrt{PS}$, and continue with the next factors to the left. Keep a correct account of the indices as described on page 8, and afterwards extract the root, as described on page 18.

Example 33. If the air pumps are double and direct action, with a diameter $D=11$ inches, what will be the diameter d of the valves, when the vacuum is $p=45$ pounds per square inch?

$$d = \frac{D \sqrt{n s}}{11 \sqrt[4]{p}} = \frac{11 \sqrt{50 \times 2.415}}{11 \sqrt[4]{4.5}} = 7.54 \text{ inches.}$$

Example 34. What will be the proper pitch of a 12 feet 9 inches propeller, geared from the steam engine, to run $n=36$ revolutions when the steam engine makes $n'=16$ revolutions?

$$P = 2.5 D \sqrt{\frac{n'}{n}} = 2.5 \times 12.75 \sqrt{\frac{16}{36}} = 21.5.$$

Example 35. What will be the slip of the propeller, the vessel being the same as in the preceding examples?

$$S = \frac{2.5}{D^3} \sqrt{\pi^2 D^2 + P^2} (\overline{Q}^2 + 2 \sqrt[3]{Q^2}) =$$

$$\frac{2.5}{12.75^3} \sqrt{3.14^2 \times 12.75^2 + 21.5^2} (300 \times 2 \sqrt[3]{875}) = 19 \text{ per cent.}$$

Example 36. What will be the pitch of a screw propeller $D=13$ feet 8 inches in diameter, the angle of the blades at the periphery being $V=52^\circ 40'$?

$$P = \pi D \frac{\cos. V}{\sin. V} = 3.14 \times 13.666 \times \frac{\cos. 52^\circ 40'}{\sin. 52^\circ 40'} = 32.73.$$

TRIAL TRIP WITH A STEAMSHIP (PROPELLER) ON THE RIVER DELAWARE.

Example 37. When passing Philadelphia, the steamer passed

South street at 1h, 3', 30" o'clock.

Vine street at 1h, 9', 40" "

in a time of $t=0h, 6', 10''$

the vessel run 5351 feet, which is the distance between South and Vine streets. What was the speed of the vessel in statute miles per hour?

$$M = \frac{5351 \times 60}{6,166 \times 5280} = 9.88 \text{ miles.}$$

The propeller made $n=56$ revolutions per minute, with a pitch $P=24$ feet. What will be the slip of the propeller?

$$\text{Slip } S = \frac{n P t - d^3}{n P t} = \frac{56 \times 24 \times 6,166 - 5351}{56 \times 24 \times 6,166} = 0,355$$

or $35\frac{1}{2}$ per cent. The propeller was 8 feet 4 inches in diameter $=D$. Area $A=54.6$ square feet. What is the power of the steam engine?

$$H = \frac{A n^2}{1950} \sqrt{P S} = \frac{54.6 \times 56^2}{1950} \sqrt{24 \times 0.355} = 257 \text{ horses.}$$

The cylinders were in diameter $d=30$ inches, and $a=706,86$; stroke $s=25$ inches; effectual pressure per square inch $p=36$ pounds; all the frictions and working the pumps being $f=30$ per cent. What will then be the power of the steam engine?

$$H = \frac{a p s n}{12 \times 33000} 4 (1-f) = \frac{706.86 \times 36 \times 26 \times 56}{12 \times 33000} 4 (1-0.30) = 262 \text{ horses.}$$

ERRATA.

On page eight, nineteenth line from the top, instead of "for multiplication, the indices are to be added. In division and subtraction," &c., *Read* "for multiplication add the factors' indices, the sum is the index for the product; and for division, subtract the index for the divisor from the index for the dividend; the remainder is the index for the quotient. When the operations are finished," &c.

NOTICE.

At as early a period as my time will allow, another calculating machine will be furnished, with instructions, designed and engraved *expressly for Navigators*. To that instrument I would particularly call their attention. By it they will be enabled, in an instant, to calculate their position at sea, latitude, longitude, courses, &c., without previous knowledge of the complicated study of navigation.

J. W. N.

